Bianchi Type-I Universe in *f(T)* Theory of Gravity-I

M.V.Dawande[1], S.S.Khan[2]

[1]Bhartiya Mahavidyalaya, Amravati, MAHARASHTRA, INDIA.

[2]P. R. Pote (Patil) College of Engineering and Management, Amravati, MAHARASHTRA, INDIA.

Abstract:

The homogeneous Bianchi type -I model in the framework of f(T) gravity has been investgated. We have used equation of state parameter, energy density and power law volumetric expansion to obtain the solutions of field equations. Some models have been constructed to examine the behavior of accelerating universe.

Keywords: *Biianchi type I Universe, f(T) theory of gravity, Equation of state parameter, Power law.*

[1] Introduction :

According to many observations and theoretical facts it seems that the universe is in the phase of accelerated expansion (Riess *et al.* 1998; Perlmutter *et al*. 1997, 1998, 1999; Spergel *et al*. 2007). The supernova experiments suggests that the universe was filled with dark energy and dark matter (Carmeli 1996, Bennett et al. 2003, Riess *et al.*2004, Spergel *et al*. 2007). Their presence in the universe is one of the puzzles of theoretical physics and studied with many alternative modified theories of gravity. One of the modified theory of gravity is the *f(R)* theory of gravity which is considered to be the most suitable theory due to its cosmological importance. *F(R)* theory of gravity (Nojiri and Odintsov 2007) gives an clear coalition of early time inflation and late time acceleration.

Another interesting modified theory of gravity is *f(T)* theory of gravity which has recently received considerable attention (Ferraro and Fiorini 2007, 2008; Bengochea and Ferraro 2009) which is based on the idea of "teleparallelism" which uses the Weitzenbock connection that has no curvature but only torsion. It is interesting to note that their equations of motion are always of second order in disparity with GR where the field equations are fourth order equations (Sharif and Shamir 2009; Sharif and Kausar 2010). Bamba *et al.* (2011) discussed different *f(T)* models to investigate the cosmological evolution of EoS parameter for dark energy. Due to spatially homogeneous and anisotropic nature, many authors have studied Bianchi Type-I spacetime in different context. Kumar and singh (2007,2008) investigated the solutions of the field equations by considering B-I universe model. Sharif and Rani (2011) studied the accelerated expansion of the universe by considering Bianchi Type-I universe.

The paper is planned as follows: Section (2) consists of basics of *f(T)* gravity. Section (3) provides the solution of field equations for B-I universe. Section (4) consists of construction of different *f(T)* models. Finally, section (5) comprises with the concluding remarks.

[2] $f(T)$ gravity and its field equations :

The modified teleparallel action for *f(T)* gravity is given by (Bamba *et al.* 2011)

$$I=\frac{1}{16\pi G}\int d^4xe(T+f(T)+L_m) \qquad (1)$$

where $e=\sqrt{-g}$, L_m stands for the matter Lagrangian and $f(T)$ is a general differentiable function of torsion.

The teleparallel Lagrangian density is described by the torsion scalar T, as

$$T=S_\rho{}^{\mu\nu}T^\rho{}_{\mu\nu}. \qquad (2)$$

The torsion, antisymmetric and contorsion tensors are defined, respectively by

$$T^{\rho}{}_{\mu\nu} = \Gamma^{\rho}{}_{\nu\mu} - \Gamma^{\rho}{}_{\mu\nu} = h_i^{\rho}(\partial_{\mu} h_{\nu}^i - \partial_{\nu} h_{\mu}^i), \quad (3)$$

$$S_{\rho}{}^{\mu\nu} = \frac{1}{2}\left(K^{\mu\nu}{}_{\rho} + \delta_{\rho}^{\mu} T^{\theta\nu}{}_{\theta} - \delta_{\rho}^{\nu} T^{\theta\mu}{}_{\theta}\right). \quad (4)$$

$$K^{\mu\nu}{}_{\rho} = -\frac{1}{2}\left(T^{\mu\nu}{}_{\rho} - T^{\nu\mu}{}_{\rho} - T_{\rho}{}^{\mu\nu}\right). \quad (5)$$

In the teleparallleism, orthogonal tetrad components $h_i\left(x^{\mu}\right)$ are used, where an index i runs over 0,1,2,3 for the tangent space at each point x^{μ} of the manifold. Their relation with metric $g_{\mu\nu}$ is given by

$$g_{\mu\nu} = \eta_{ij} h_{\mu}^i h_{\nu}^j, \quad (6)$$

where $\eta_{ij} = diag(1,-1,-1,-1)$ is the Minkowski metric for the tangent space satisfying the following properties (Ferraro and Fiorini 2007; Hayashi and Shirafuji 1979)

$$h_{\mu}^i h_j^{\mu} = \delta_j^i, \; h_{\mu}^i h_i^{\nu} = \delta_{\mu}^{\nu}. \quad (7)$$

The variation of equation (1) with respect to tetrad h_{μ}^i leads to the following field equations

$$\left[e^{-1}\partial_{\mu}\left(eS_i{}^{\mu\nu}\right) - h_i^{\lambda} T^{\rho}{}_{\mu\lambda} S_{\rho}{}^{\nu\mu}\right]\left(1 + f(T)\right) + S_i{}^{\mu\nu}\partial_{\mu}(T) f_{TT} + \frac{1}{4} h_i^{\nu}\left[T + f(T)\right] = \frac{1}{2} k^2 h_i^{\rho} T_{\rho}^{\nu}, \quad (8)$$

where $S_i{}^{\mu\nu} = h_i^{\rho} S_{\rho}{}^{\mu\nu}$, $k^2 = 8\pi G$, $f_T \equiv \dfrac{df}{dT}$,

and the energy momentum tensor is given by

$$T_{\rho}^{\nu} = diag(\rho_M, -P_M, -P_M, -P_M), \quad (9)$$

where ρ_M is the energy density and P_M is the pressure of matter inside the Universe.

[3] Metric and Solutions :

The line element for spatially homogeneous and anisotropic Bianchi-I universe is

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)dy^2 - C^2(t)dz^2, \quad (10)$$

where A, B and C are functions of cosmic time t .

Using equations (1) and (10), we obtained the tetrad components as

$$h_{\mu}^i = diag(1, A, B, C),$$

$$h_i^{\mu} = diag(1, A^{-1}, B^{-1}, C^{-1}). \quad (11)$$

After substituting Equations (3) and (4) in equation (2) the torsion T for B-I becomes

$$T = -2\left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA}\right) \quad (12)$$

The average scale factor R , the anisotropy parameter Δ and the mean Hubble parameter H are given by

$$R = (ABC)^{\frac{1}{3}}, \quad \Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2,$$

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \quad (13)$$

where H_i are the directional Hubble paramerters in x, y and z direction respectively given as,

$$H_1 = \frac{\dot{A}}{A}, \quad H_2 = \frac{\dot{B}}{B}, \quad H_3 = \frac{\dot{C}}{C}.$$

For $i = 0 = \nu$ and $i = 1 = \nu$, we get the below field equations

$$T + f(T) - 4\left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA}\right)\left(1 + f(T)\right) = 2k^2\rho_M \quad (14)$$

$$\begin{aligned}
&2\left(2\frac{\dot{B}\dot{C}}{BC} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC}\right)\left(1 + f(T)\right) \\
&-4\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\left[\begin{array}{l}\frac{\dot{A}}{A}\left(\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} + \frac{\ddot{C}}{C} - \frac{\dot{C}^2}{C^2}\right) + \frac{\ddot{A}}{A}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{\dot{B}}{B}\left(\frac{\ddot{C}}{C} - \frac{\dot{C}^2}{C^2}\right) + \\ \frac{\dot{C}}{C}\left(\frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2}\right) - \frac{\dot{A}^2}{A^2}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\end{array}\right] f_{TT} - (T + f) \\
&= 2k^2 P_M
\end{aligned} \quad (15)$$

By using the power-law volumetric expansion, we get

$$V = C_1 t^{3m}, \quad (16)$$

where C_1 is arbitrary constant of integration.

Using this condition, we have found

$$\Delta = 0 \quad (17)$$

For $\Delta = 0$ the isotropic behavior of the expanding Universe is obtained and by using this condition the above field equation becomes,

$$H^2 = \frac{-8\pi G\rho_M}{9} + \frac{f}{18} + \frac{Tf_T}{9} \quad (18)$$

$$\left(H^2\right)' = \frac{4\pi GP_M}{\left(3 - \frac{1}{m}\right)\left(1 + f_T\right)} + \frac{T + f(T)}{\left(12 - \frac{4}{m}\right)\left(1 + f_T\right)} \quad (19)$$

For $T+f(T)=T$ the above equations reduces to field equations as,

$$H^2 = \frac{-8\pi G}{9}\left(\rho_M + \rho_{DE}\right) \qquad (20)$$

$$\left(H^2\right)' = \frac{4\pi G}{\left(3-\frac{1}{m}\right)}\left(P_M + P_{DE}\right) + \frac{T}{\left(12-\frac{4}{m}\right)} \qquad (21)$$

Here we consider only non-relativistic matter with pressure zero i.e. $P_M = 0$.

The energy density and pressure of the effective dark energy can be described by comparing Eq.(18) with Eq.(20) and Eq.(19) with Eq.(21) as,

$$\rho_{DE} = \frac{-1}{16\pi G}\left(f + 2Tf_T\right) \qquad (22)$$

$$P_{DE} = \frac{\left(3-\frac{1}{m}\right)}{4\pi G}\left(\frac{f(T)-Tf_T}{\left(12-\frac{4}{m}\right)\left(1+f_T\right)}\right) \qquad (23)$$

The energy conservation equations corresponding to standard matter and DE are

$$\dot{\rho}_M + 3H\rho_M = 0,$$

$$\dot{\rho}_{DE} + 3H\rho_{DE}(1+\omega_{DE}) = 0$$

Here, $\omega_{DE} = \frac{P_{DE}}{\rho_{DE}}$ is the effective EoS parameter for $f(T)$ gravity.

The equation of state for dark energy is

$$\omega_{DE} = -\left[\frac{f(T)-Tf_T}{\left(1+f_T\right)\left(f+2Tf_T\right)}\right] \qquad (24)$$

[4] Construction of some $f(T)$ models :

In this section, we have proposed three *f(T)* models, named as Exponential Model, Logarithmic Model and Combined Exponential and Logarithmic Model, which can realize the crossing of the phantom divide line for the effective equation of state.

[4.1] EXPONENTIAL $f(T)$ MODEL :

We consider the exponential *f(T)* model [Linder 2010; Bamba *et al*. 2011]

$$f(T) = \alpha T\left(1 - e^{\frac{PT_0}{T}}\right) \qquad (25)$$

where $\alpha = -\frac{1-\Omega_M^{(0)}}{1-(1-2p)e^p}$, p is a constant.

We take initial value $T_0 = T(z=0)$ of the torsion and the red shift *z* is given by

$$Z = \frac{1}{t^m} - 1 \qquad (26)$$

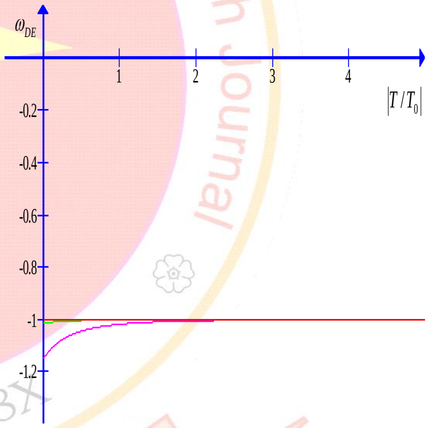
At present $\Omega_m^{(0)} \equiv \rho_M^{(0)}/\rho_{crit}^{(0)} = 0.26$, where $\rho_m^{(0)}$ is the energy density of non-relativistic matter defined by [Komatsu *et al.* 2011]

$\rho_{crit}^{(0)} = \frac{3H_0^2}{8\pi G}$ is the critical density [Bamba *et al.* 2010] with current Hubble parameter $H_0 = 1$ [Sharif and Jawad 2012]. Here equation (26) has only parameter *p* if value of $\Omega_M^{(0)}$ is known.

The EoS parameter in terms of $|T/T_0|$ is given by

$$\omega_{DE} = \frac{PT_0 e^{\frac{PT_0}{T}}}{TE} \qquad (27)$$

where



$$E = \left[1 + \alpha\left(1 - e^{\frac{PT_0}{T}}\right) + \frac{\alpha PT_0}{T}e^{\frac{PT_0}{T}}\right]\left[3\left(1 - e^{\frac{PT_0}{T}}\right) + \frac{2PT_0 e^{\frac{PT_0}{T}}}{T}\right]$$

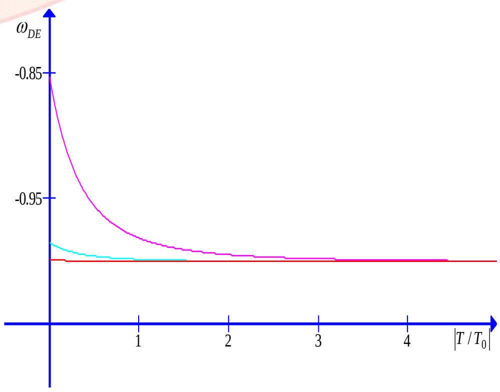


Fig.1 Plot of ω_{DE} versus $|T/T_0|$ with m=2 for exponential *f(T)* model. In left graph, $|p| = 0.1(pink), 0.01(green), 0.001(red)$ and in right graph $|p| = -0.1(pink), -0.01(green), -0.001(red)$ respectively.

The behavior of ω_{DE} as a function of $|T/T_0|$ is graphically represented in fig.1. From the nature of graph, it can be easily verified that for any value of *p*, ω_{DE} approaches to -1 but does not cross the phantom divide line $(\omega_{DE} = -1)$. Therefore, as $|T/T_0| \to \infty$, the universe stays in the DE era.

Putting Eq.(16) in Eq.(12) we obtain a torsion scalar which is a function of redshift *z*.

$$t^m = \left(\frac{-6m^2}{T}\right)^{\frac{m}{2}} \tag{28}$$

$$T = -6m^2(1+Z)^{\frac{2}{m}} \quad , \quad T_0 = -6m^2 \tag{29}$$

Using Eq.(29), Eq.(27) becomes,

$$\omega_{DE} = \frac{Pe^{P(1+Z)^{\frac{-2}{m}}}}{D(1+Z)^{\frac{2}{m}}} \tag{30}$$

Where,

$$D = \left[1 + \alpha\left(1 - e^{P(1+Z)^{\frac{-2}{m}}}\right) + \alpha P(1+Z)^{\frac{-2}{m}} e^{P(1+Z)^{\frac{-2}{m}}}\right]\left[3\left(1 - e^{P(1+Z)^{\frac{-2}{m}}}\right) + 2P(1+Z)^{\frac{-2}{m}} e^{P(1+Z)^{\frac{-2}{m}}}\right]$$

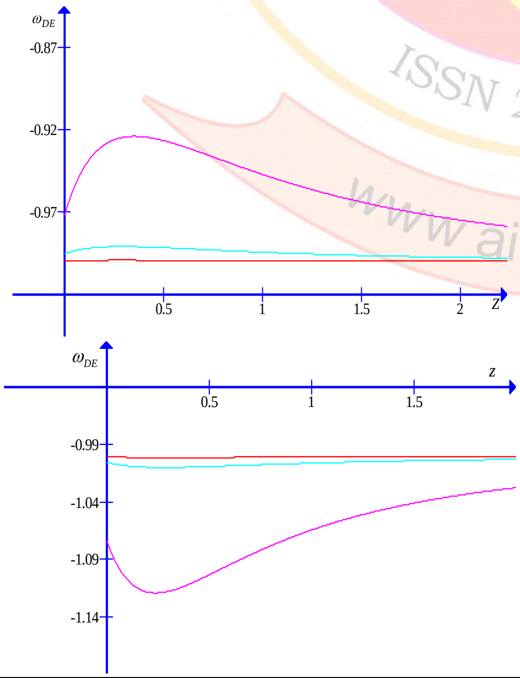
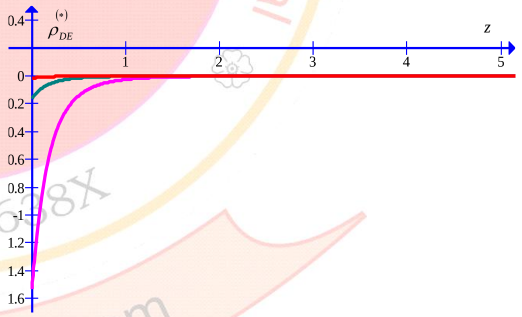


Fig.2 Plot of ω_{DE} versus redshift z for exponential *f(T)* model with m=2 and the values of $|p| = 0.1, 0.01, 0.001$ and $|p| = -0.1, -0.01, -0.001$ respectively.

The behavior of ω_{DE} versus redshift *z* is graphically shown in fig.2. which depict the equation of state for dark energy ω_{DE} as a function of the redshift *z* for $|p| = 0.1, 0.01, 0.001$ and $|p| = -0.1, -0.01, -0.001$ respectively. For the positive values of *p*, ω_{DE} decreases and approaches to -1 but does not cross the phantom divide line for z approaches to ∞. Hence, the universe remains in the non-phantom phase(quintessence). For the negative values of p, ω_{DE} which was initially less than -1, increases and approaches to -1 without crossing the phantom divide line. Initially when z approaches to ∞ it gives the phantom phase of the universe . Thus, for both cases, the universe persist in the DE era.

$$\overset{(*)}{\rho}_{DE} = \frac{3m^2(1+Z)^{\frac{2}{m}}\alpha}{8\pi G \overset{(0)}{\rho}_{DE}}\left[3\left(1 - e^{P(1+Z)^{\frac{-2}{m}}}\right) + \frac{2Pe^{P(1+Z)^{\frac{-2}{m}}}}{(1+Z)^{\frac{2}{m}}}\right] \tag{31}$$

where $\overset{(0)}{\rho}_{DE} = 0.74 \overset{(0)}{\rho}_{crit}$ (Bamba *et al*. 2011).



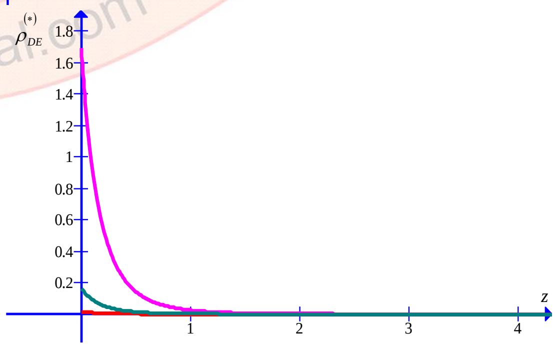


Fig.3 Plot of $\overset{(*)}{\rho}_{DE}$ versus redshift *z* for exponential *f(T)* model keeping the same parameters as in Fig. 2

The evolution of $\overset{(*)}{\rho}_{DE}$ in terms of redshift z for positive and negative values is shown in Fig.3. A slight increment in $\overset{(*)}{\rho}_{DE}$ can be seen for the positive values of *p* and smaller values of *z*, whereas it takes a constant value, for larger values of *z*. However for negative values of *p*, $\overset{(*)}{\rho}_{DE}$ decreases initially with increasing z and reaches to constant value as z approaches to ∞. Here we can note that for both cases of *p*, $\overset{(*)}{\rho}_{DE}$ attains different values at $z=0$.

Now for phantom and non-phantom phases we check the relevancy of the exponential $f(T)$ model by using an approximate method.

From Eq. (25) we have obtained

$$f_T=\alpha\left(1-e^{\frac{PT_0}{T}}+\frac{PT_0}{T}e^{\frac{PT_0}{T}}\right) \qquad (32)$$

By considering $X=pT_0/T$, $T_0/T\le 1$ in Eqs.(25) and (32), it follows that (Bamba *et al.* 2010b)

Assuming $X=\frac{PT_0}{T}$, $\frac{T_0}{T}\le 1$

$$\frac{f}{T}\approx-\alpha\left(X+\frac{X^2}{2}\right),\ f_T\approx\frac{\alpha X^2}{2} \qquad (33)$$

ω_{DE} takes the following form after putting these values in Eq.(24),

$$\omega_{DE}\approx-\frac{(1+X)}{\left(1+\frac{\alpha X^2}{2}\right)\left(1-\frac{X}{2}\right)} \qquad (34)$$

We consider $\alpha\sim O(1)$. It seems that the dependency of EoS parameter is on the sign of *p* and on *X*. For the positive value of *X* we get $\omega_{DE}\rangle-1$ which gives the non-phantom phase of the universe. For the negative value of *X*, we obtain $\omega_{DE}\langle-1$ which implies the phantom phase of the universe.

[4.2] LOGARITHMIC $f(T)$ MODEL :

We take the logarithmic $f(T)$ model as (Bamba *et al.* 2011)

$$f(T)=\beta T_0\left(\frac{qT_0}{T}\right)^{\frac{-1}{2}} In\left(\frac{qT_0}{T}\right) \qquad (35)$$

Where,

$$\beta\equiv\frac{1-\Omega_M^{(0)}}{2q^{\frac{-1}{2}}},\qquad q>0$$

Same as the exponential $f(T)$ model, in logarithmic model also only the parameter *q* is involved if value of $\Omega_M^{(0)}$ is given.

The EoS parameter which is independent of *q* is given as

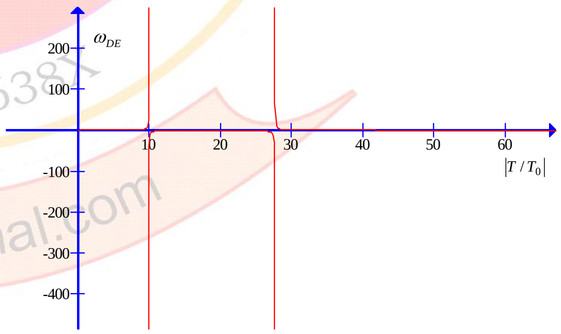
$$\omega_{DE}=-\left[\frac{\frac{1}{2}\log\left(\frac{T_0}{T}\right)+1}{\left(2+\left(\frac{1-\Omega_M^{(0)}}{2}\right)\left(\frac{T_0}{T}\right)^{\frac{1}{2}}\log\left(\frac{T_0}{T}\right)-\left(1-\Omega_M^{(0)}\right)\left(\frac{T_0}{T}\right)^{\frac{1}{2}}\right)\left(\log\left(\frac{T_0}{T}\right)-1\right)}\right] \qquad (36)$$

By using Eq.(29) in Eq.(36), ω_{DE} takes the following form

Since $T=-6m^2(1+Z)^{\frac{2}{m}}$ &

$$\frac{T_0}{T}=(1+Z)^{\frac{-2}{m}}$$

$$\omega_{DE}=-\left[\frac{1+\frac{1}{2}\log(1+Z)^{\frac{-2}{m}}}{\left[2+\left(\frac{1-\Omega_M^{(0)}}{2}\right)(1+Z)^{\frac{-1}{m}}\log\frac{1}{(1+Z)^{\frac{2}{m}}}-\left(1-\Omega_M^{(0)}\right)(1+Z)^{\frac{-1}{m}}\right]\left[\log\frac{1}{(1+Z)^{\frac{2}{m}}}-1\right]}\right] \qquad (37)$$



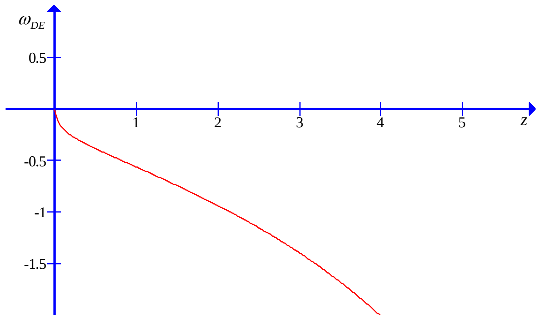


Fig.4 Plot of ω_{DE} versus $|T/T_0|$ in the left side and ω_{DE} versus *z* in right side for logarithmic *f(T)* model with m=2 and the values of $|p| = 0.1, 0.01, 0.001$ and $|p| = -0.1, -0.01, -0.001$ respectively.

Fig. 4 shows the behavior of ω_{DE} and $|T/T_0|$. Left graph shows that ω_{DE} takes negative value as $|T/T_0| \to \infty$. Initially the model displays the properties of both matter and radiation. After a while, the universe enters in DE era. So we can say that as the time passes, the universe stays in non-phantom phase. Right graph show the same behavior as that of the exponential *f(T)* model and hence the universe stays in the non-phantom phase $(\omega_{DE} \rangle -1)$.

[4.3] COMBINED EXPONENTIAL AND LOGARITHMIC $f(T)$ MODEL:

Now we consider the combination of both exponential and logarithmic $f(T)$ models as (Bamba *et al*. 2011)

$$f(T) = \gamma\left[T_0\left(\frac{uT_0}{T}\right)^{\frac{-1}{2}} In\left(\frac{uT_0}{T}\right) - T\left(1 - e^{\frac{uT_0}{T}}\right)\right]$$

(38)

The only parameter in this model is the positive constant *u*. The EoS parameter for DE is given by

$$\omega_{DE} = -\frac{1}{I}\left[\frac{1}{u}\sqrt{\frac{uT_0}{T}}\left(\frac{1}{2}\log\left(\frac{uT_0}{T}\right) - 1\right) - 2\left(1 - e^{\frac{uT_0}{T}}\right) - e^{\frac{uT_0}{T}}\frac{uT_0}{T}\right] \quad (39)$$

Where

$$I = \left[1 + \gamma\left\{\frac{1}{u}\sqrt{\frac{uT_0}{T}}\left(\frac{1}{2}\log\left(\frac{uT_0}{T}\right) - 1\right) - \left(1 - e^{\frac{uT_0}{T}}\right) - e^{\frac{uT_0}{T}}\frac{uT_0}{T}\right\}\right]$$
$$\left[\frac{2}{u}\sqrt{\frac{uT_0}{T}}\left(\log\left(\frac{uT_0}{T}\right) - 1\right) - 3\left(1 - e^{\frac{uT_0}{T}}\right) - 2e^{\frac{uT_0}{T}}\frac{uT_0}{T}\right]$$

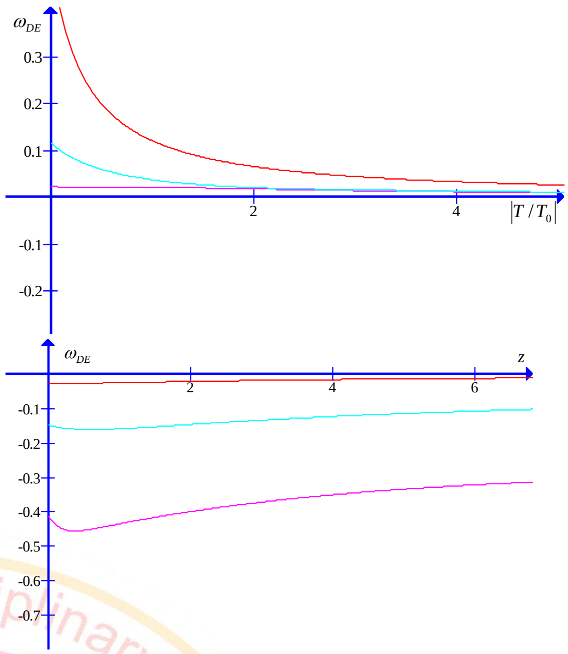


Fig.5 Plot of ω_{DE} versus $|T/T_0|$ in the left side and ω_{DE} versus z in right side in the combined exponential and logarithmic model with m=2 and the values of $|p| = 0.1, 0.01, 0.001$ and $|p| = -0.1, -0.01, -0.001$ respectively.

Left graph of Fig. 5 shows that in the beginning the universe lies in the non-phantom phase as ω_{DE} acts as a function of $|T/T_0|$. It can be seen that ω_{DE} decreases with increase in $|T/T_0|$ and crosses the phantom divide line. Again within some finite time ω_{DE} crosses the phantom line and takes constant value as $|T/T_0|$ increases.

Putting Eq.(29) in Eq.(39), ω_{DE} in terms of *z* becomes

$$\omega_{DE} = -\frac{1}{F}\left[\frac{1}{u}\sqrt{u(1+Z)^{\frac{-2}{m}}}\left(\frac{1}{2}\log\left(u(1+Z)^{\frac{-2}{m}}\right) - 1\right) - 2\left(1 - e^{u(1+Z)^{\frac{-2}{m}}}\right) - e^{u(1+Z)^{\frac{-2}{m}}}u(1+Z)^{\frac{-2}{m}}\right] \quad (40)$$

Where

$$F = \left[1 + \gamma\left\{\frac{1}{u}\sqrt{u(1+Z)^{\frac{-2}{m}}}\left(\frac{1}{2}\log\left(u(1+Z)^{\frac{-2}{m}}\right) - 1\right) - \left(1 - e^{u(1+Z)^{\frac{-2}{m}}}\right) - e^{u(1+Z)^{\frac{-2}{m}}}u(1+Z)^{\frac{-2}{m}}\right\}\right]$$
$$\left[\frac{2}{u}\sqrt{u(1+Z)^{\frac{-2}{m}}}\left(\log\left(u(1+Z)^{\frac{-2}{m}}\right) - 1\right) - 3\left(1 - e^{u(1+Z)^{\frac{-2}{m}}}\right) - 2e^{u(1+Z)^{\frac{-2}{m}}}u(1+Z)^{\frac{-2}{m}}\right]$$

Right graph of Fig. 5 shows that for different values of u, the universe stays in the phantom phase initially. But as z increases , ω_{DE} crosses the phantom divide line and enters the non-phantom phase and approaches to a constant value. It is interesting to note that the behavior of the combined *f(T)* models is similar to quintom model (Khatua *et al*. 2011).

[5]Conclusion:

In this paper, we have studied the different phases of the B-I universe in the context of *f(T)* gravity. Through graphical representation, phantom and non-phantom phases of the B-I universe is examined. It is noted that

(i) For exponential model, the phase of the B-I universe depends on the sign of the parameter p i.e. for $p \rangle 0$ and $p \langle 0$ the universe is always in non-phantom and phantom phase without crossing the phantom divide line.

(ii) For logarithmic model, the crossing of phantom divide line does not exist.

(iii) In the combined exponential and logarithmic model, the crossing of phantom divide can be seen.

References

[1] Riess, A. G., *et al*.: Astron. J. **116**, 1009 (1998).

[2] Perlmutter, S., *et al*.: Astrophys. J. **483**, 565 (1997).

[3] Perlmutter, S., *et al*.: Nature **391**, 51 (1998).

[4] Perlmutter, S., *et al*.: Astrophys. J. **517**, 565 (1999).

[5] Spergel, D. N., *et al*.: Astrophys. J. Suppl. **170**, 377 (2007).

[6] Carmeli, M.: Commun. Theor. Phys. **5** (1996) 159.

[7] Riess, A. G., *et al*.: Astrophys. J. **607**, 665 (2004).

[8] Betnnett, C., *et al*.: Astrophys. J. Suppl. **148**, 1 (2003).

[9] Nojiri, S., Odintsov, S. D.: Int. J. Geom. Meth. Mod.Phys. **4**, 115 (2007).

[10] Ferraro, R., Fiorini, F.: Phys. Rev. D **75**, 084031 (2007).

[11] Ferraro, R., Fiorini, F.: Phys. Rev. D **78**, 124019 (2008).

[12] Bengochea, G. R. and Ferraro, R.: Phys. Rev. **D79**, 124019 (2009).

[13] Sharif, M., Shamir, M.F.: Class. Quantum Gravity **26**, 235020 (2009).

[14] Sharif, M., Kausar, H. R.: Mod. Phys. Lett. A **25**, 3299 (2010).

[15] Linder, E. V.: arXiv: **1005**. 3039v2 [astro-ph.co] (2010).

[16] Bamba, K. *et al*.: J. Cosmol. Astropart. Phys. **01**, 021 (2011).

[17] Kumar, S. and Singh, C.P.: Astrophys. Space Sci. **312** (2007) 57.

[18] Kumar, S. and Singh, C.P.: Int. J. Theor. Phys. **47** (2008) 1722.

[19] Sharif, M., Rani, S.: Mod Phys. Lett. A **26**, 1657 (2011a).

[20] Hayashi, K. and Shirafuji, T.: Phys. Rev. D **19**, 3524 (1979).

[21] Komatsu, E., *et al*.: Astrophys. J. Suppl. **192**, 18 (2011).

[22] Bamba, K., *et al*.: JCAP **11**, 001 (2010a).

[23] Sharif, M., Jawad, A.: Astrophys. Space Sci. **337**, 789 (2012).

[24] Khatua, P. B., *et al*.: arXiv: **1105**.3393 (2011).